A 3D-BPP approach for optimising stowage plans and terminal productivity

Anna Sciomachen *, Elena Tanfani

Department of Economics and Quantitative Methods (DIEM), University of Genova, Via Vivaldi 5, 16126 Genova, Italy

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Abstract

This paper addresses the problem of determining stowage plans for containers in a ship, that is the so-called master bay plan problem (MBPP).

MBPP is NP-complete [Botter, R.C., Brinati, M.A., 1992. Stowage container planning: A model for getting an optimal solution. IFIP Transactions B (Applications in Technology) B-5, 217–229; Avriel, M., Penn, M., Shpirer, N., 2000. Container ship stowage problem: Complexity and connection to the colouring of circle graphs. Discrete Applied Mathematics 103, 271–279]. We present a heuristic method for solving MBPP based on its relation with the three-dimensional bin packing problem (3D-BPP), where items are containers and the only bin is the ship. We look for stowage plans that take into a proper account structural and operational constraints, related to both the containers and the ship, and maximise some important terminal performance indexes, such as the effective and mean net crane productivity.

Our aim is to evaluate how stowage plans can influence the performance of the quay. A validation of the proposed approach with some test cases related to containership docks at the port of Genoa (Italy) is given. The results of real instances of the problem and the comparison with a validated heuristic for MBPP, show the effectiveness of the proposed approach in producing stowage plans that minimise the total loading time and allow an efficient use of the quay equipment.

Keywords: Three-dimensional bin packing problem; Ship loading; Terminal efficiency; Heuristic methods

1. Introduction and literature review

During the past 30 years the container handling revolution has increased the efficiency of worldwide trade. The container is a universal transportation method capable of being moved by sea, road or rail with relative ease. With the increasing level of world trade and the corresponding increase in size of containerships (up to around 8000 containers by the year 2000) one of the main areas for cost and efficiency gains now occurs in port whilst performing container loading, unloading and arrangement. The shorter the turnaround time, the greater the financial savings. Thus any methods to improve the process would be significant.
The loading or stowage of a containership is one of the problems that has to be solved daily by any company which manages a terminal container (Thomas, 1989). In the past stowage plans for containers were performed by the Captain of the ship; today, the maritime terminal has to establish the master bay plan, in accordance with the stowage instructions of the ship co-ordinator representing the company holding the ship.

The ship planning problem involves different objectives, such as optimal space allocation, optimal synchronisation among dispatching operations and minimisation of the berthing time (Atkins, 1991).

Formally, the stowage planning problem, that is commonly denoted the master bay plan problem (MBPP), consists in determining how to stow a set \( C \) of \( n \) containers of different type into a set \( S \) of \( m \) available locations of a containership, with respect to some structural and operational constraints, related to both the containers and the ship, while minimising the total stowage time, which is given by the time required for loading all containers on board plus the shifting cost due to the removal of containers.

The following points outline the main constraints that must be considered for the stowage planning process for an individual port (Ambrosino et al., 2004).

- The constraints related to the structure of the ship are focused on the type, size and weight of the containers to be loaded. For the definition of the stowage plan we consider only standard containers, having exterior dimensions conforming to ISO standards of 20 and 40 ft long, 8 ft 6 in. high and 8 ft depth. In particular, containers of 40 foot (in the following denoted by 40') require two contiguous locations of 20 foot (in the following denoted by 20'). Because of the numerical cell system usually chosen by any maritime company (see Fig. 1), 40' containers can be located only in even bays; consequently, those odd bay cells in the same row and tier, corresponding to the even bay cells used for the stowage of 40' containers, become unavailable for stowing 20' containers. Moreover, 20' containers cannot be stacked above 40' ones, and both 20' and 40' containers cannot be stacked above empty cells.
- Weight constraints force the weight of a stack of containers to be less than a given tolerance value; moreover, the weight of a container located in a tier cannot be greater than the weight of the container located below it in the same row and bay.
- As for the type of containers, locations of reefer containers are defined in advance by the ship co-ordinator, in fact their location is constrained to be in a designated part of the ship (such a specific bay) where power points are available in order to maintain the required temperature during transportation. The locations of hazardous containers are predetermined too by the harbour-master’s office, which authorises their loading.
- Operational and security constraints are related to the weight distribution on the ship. In particular, after any loading/unloading operations different kinds of equilibrium have to be checked,
namely: cross equilibrium, that is the weight on the right side of the ship must be equal, within a given tolerance, to the weight on the left side of the ship; horizontal equilibrium, that is the weight on the stern must be equal, within a given tolerance, to the weight on the bow; vertical equilibrium, that is the weight on each tier must be greater than the weight on the tier immediately over it.

- Finally, destination constraints give a general rule which suggests loading first those containers having as destination the final stop of the ship and consequently load last those containers that have to be unloaded first. In fact, commonly a vessel will call at several ports on a circular journey. At each port containers for that port are unloaded, and this may involve removing and restowing containers bound for later ports which have been placed on top of others bound for the current port. The number of these container restows should be minimised in order to save port time and hence costs. This is shown in Fig. 2, where all black containers need to be unloaded but in order to do this three white ones have to be removed; such unloaded white containers are the restows.

The present combinatorial optimisation problem has been proved to be NP-complete (Botter and Brinati, 1992; Avriel et al., 2000). The scale of the problem is enormous. For a 2000 TEUs (twenty-foot equivalent unit) containership, such as the one tackled here, the number of possible stowage configurations is approximately $3.3 \times 10^{5735}$ and as the size of containerships increases the reliance on manual systems becomes increasingly difficult and more costly (Saginaw and Parakis, 1989).

An early attempt at this problem explained by Shields (1984), uses a combination of simulation using Monte-Carlo techniques and human interaction to provide possible vessel plans. Since then further investigations have been carried out (Ratcliffe and Sen, 1987; Saginaw and Parakis, 1989) using expert systems and rule-based techniques to aid the stevedore in finding suitable configurations.

Rule-based decision systems for dealing with MBPP are presented in Ambrosino and Sciomachen (1998), where a constraints satisfaction approach is used for defining and characterising the space of feasible solutions without employing an objective function to optimise, and in Wilson and Roach (2000), where the potential of applying the theory of artificial intelligence to cargo stowage problems is explored.

Dubrovsky et al. (2002) use a genetic algorithm for minimising the number of container movements in the stowage planning problem, while being able to include with appropriate constraints some ship stability criteria. The authors significantly reduce the search space using a compact and efficient encoding scheme and obtain good solution for instances of 1000 TEUs ships.

The application of Mathematical Programming models to MBPP has been investigated in Avriel and Penn (1993), Botter and Brinati (1992), Chen et al. (1995) and Imai et al. (2002), but many simplification hypotheses to the proposed Linear Programming models and Integer Programming ones make them unsuitable for practical applications.

Wilson and Roach (1999) and Wilson et al. (2001) test the application of local search algorithms and techniques based on combinatorial optimisation. In particular, the authors break the container stowage process into two phases, at a strategic and tactical planning level, respectively. In particular, they use branch and bound algorithms for solving the problem of assigning generalized containers to a bays’ block in a vessel; in the second step they find a detailed plan which assigns specific positions or locations in a block to specific containers by a tabu search algorithm. The computational experiments reported by the authors show the goodness of the
sub-optimal solutions obtained in computational time of the order of 90 minutes for instances relative to a 688 TEU ship. A similar staged approach has been followed by Ambrosino et al. (2006) that, in a first phase, split the set of bays of a containership and use a branch and bound algorithm for each subset of bays, looking successively for the global stability of the ship by performing multi-exchanges.

Martin et al. (1988) develop a heuristic algorithm for the MBPP with the aim of minimising the longitudinal movement time of the quay cranes and the total number of container restows.

Haghani and Kaisar (2001) have developed a heuristics algorithm similar to the method due to Martin et al. (1988), with the aim of minimising the total container restows in the route and taking an acceptable level of stability during the loading operations. Terno et al. (2000) consider in their analysis the container weights and the ship stability deriving the MBPP to the “multi-pallet loading problem”, without presenting adequate examples for MBPP.

In this paper, we go further in the analysis of the connection between MBPP and 3D-BPP presented in Sciomachen and Tanfani (2003), which is based on the exact branch-and-bound algorithm for 3D-BPP presented by Martello et al. (2000) and extend the procedure into a three phase approach for including the maximization of the quay equipment productivity. Note that, according to the improved typology of Cutting and Packing Problems reported in Wäscher et al. (this issue), we consider the MBPP as a three-dimensional (Orthogonal) bin packing problem.

We assume that we are not involved with the stowage of non-standard dimensions containers as well as with containers having special handling and stowage requirements (e.g., hazardous and reefer containers) and that the number of containers to load on board is not greater than the number of available locations. Moreover, we make the assumption that the ship starts its journey in the port for which we are studying the problem and successively visits a given number of other ports where only unloading operations are allowed; we can justify this assumption by remembering that we are involved with the stowage planning problem of a terminal that is not really affected by what happens in the next ports, and hence we do not interact with the ship co-ordinator.

In Section 2, we show through a simple example the main commonalities and differences between 3D-BPP and MBPP. In Section 3, we introduce the performance indexes utilised to evaluate the terminal productivity. The main steps of our algorithm for dealing with MBPP with the aim of maximising the given indexes are presented in Section 4. In Section 5 we report some computational experiments performed with real instances that compare two heuristics for the stowage plans of a ship performed with two quay cranes; the results show the effectiveness of the proposed approach in terms of goodness of the solutions, computational time and quay productivity. Finally, in Section 6 we give some concluding remarks and outlines for future work.

2. Comparison of 3D-BPP and MBPP via a simple example

As has already been said, the proposed heuristic procedure for maximising quay terminal productivity is based on the relation of MBPP with the three-dimensional bin packing problem.

Given a set of \( n \) rectangular-shaped items, each one characterised by width \( w_j \), height \( h_j \), and depth \( d_j \) (\( j \in J = \{1, \ldots, n\} \)), and an unlimited number of identical three-dimensional containers (bins) having width \( W \), height \( H \), and depth \( D \), 3D-BPP consists of orthogonally packing all items into the minimum number of bins.

In the exact branch-and-bound algorithm for 3D-BPP presented by Martello et al. (2000), the authors assume that (a) items may not be rotated, (b) items are packed with each edge parallel to the corresponding bin edge, (c) all data are positive integers such that \( w_j \leq W \), \( h_j \leq H \), and \( d_j \leq D \), \( \forall j \in J \). Those few restrictions make this approach fortunately applicable to MBPP. In particular, assumptions (a) and (b) are not to be underestimated; in fact, usually in the bin packing problem’s literature these strong restrictions are not considered and items can be rotated (see e.g., Bischoff and Mariott, 1990; Faina, 2000; Gehring et al., 1990; Mohanty et al., 1994). Note that assumption (a) is not really important in the packing practice; instead, it is absolutely necessary for the definition of stowage plans since containers have to be stowed only in one orthogonal direction.

The enumerative algorithm given in Martello et al. (2000) for 3D-BPP iteratively solves associated sub-problems in which all items of a given subset \( J' \subseteq J \) have to be packed into a single bin (if it is possible), with the aim of maximising the total volume of the packed items. In particular, a procedure,
called “main branching tree” assigns items to bins without specifying their actual position, and a branch-and-bound algorithm, called “onebin”, verifies whether a subset of items \( J' \subseteq J \) can be placed inside a single bin and, if it is the case, finds the best filling of the single bin using items belonging to \( J' \).

In Sciomachen and Tanfani (2003) the structural and operational constraints related to both the containers and the ship, that are obviously not foreseen in the 3D-BPP formulation, have been considered. In that work the goal is to minimise the total loading time, which is one of the most important productivity index of maritime terminals for being competitive. Here, we want to go further and include in the procedure for the optimal MBPP the quay side equipment with the aim of maximising the effective and mean net crane productivity index.

Our first effort has been devoted to make the mentioned exact 3D-BPP algorithm “formally” usable for the resolution of MBPP. We have implemented a procedure for expressing the input/output data in terms of containers, instead of items. In particular, this procedure gives the dimensions of the items and visualizes the co-ordinates \((x, y, z)\) of their final position, where \(x\) is the width, \(y\) is the height and \(z\) is the depth. Moreover, we have considered the possibility of giving as input the dimensions of the bin, in terms of numerical values for \(H, D\) and \(W\) (that is height, depth and width). Finally, we have imposed a priori the possible dimensions of the containers (items), that is 20’ and 40’, allowing the user of the implementation to choose the number of containers to be loaded for each size; just for testing purposes, a random selection between them is also allowed.

We divide the main bin, that is the ship, into different sections in order to be able to consider the above and below deck spaces, the bow and the stern as separate components. Note that our approach does not address the interconnecting hatch lids separating the above and below decks, which impact on the time that ships spent at port.

To analyse the commonalities and differences between MBPP and 3D-BPP, let us present a simple case study concerning the stowage plan of a prototype 80 TEUs containership, with five 20’ bays, four rows, and four tiers. \( n = 50 \) standard containers, split into 20’ and 40’, have to be loaded in the ship to the same destination; their weight ranges in three classes, namely Light (L), Medium (M) and Heavy (H), from 10 to 30 tons (see Table 1).

The solution of this simple case obtained by using the exact algorithm for the 3D-BPP is reported in Fig. 3, that depicts the corresponding master bay plan for each tier and the progressive number of each container in the inverse order of their loading, such that the last stowed container is identified by “1” and the first one by “\( n \)”. Looking at Fig. 3 we can understand how containers are positioned on board, from a 3D-BPP perspective.

We can immediately note that the proposed solution is not feasible for MBPP. In fact, first the referring 3D-BPP algorithm starts to position the biggest items from the back left bottom corner and continues to fill the bin in a vertical pattern, that is containers are stacked one above the other until the maximum height of the bin is reached; then, the bin is filled width-wise (horizontal pattern) and finally depth-wise (transversal pattern). Consequently, the total weight is concentrated near the origin of the axes, where all 40’ containers are positioned, thus compromising the stability of the ship. We also note the presence of empty spaces in the second bay \((Z = 20)\). Instead, in the case of stowage plans it would be better to follow horizontal patterns due to the stability of the ship and not allowing the presence of empty spaces between items.

Secondly, because of the characteristics of 3D-BPP, the largest items are packed first, as their placement is more difficult; this order of preference applied to our problem arises in stowing first 40’ containers and, consequently, 20’ containers are positioned over them, thus violating the size constraints (see Section 1). Note that size constraints are violated since 20’ containers are stowed above 40’ ones. Finally, in 3D-BPP the position of the items is given in a three-dimensional Cartesian space \((XYZ)\), with the origin of the axis located at the back left bottom corner, while for the definition of stowage plans we have to know the exact position of the containers according to the “numerical cell

<table>
<thead>
<tr>
<th>( n )</th>
<th>Size</th>
<th>Weight</th>
<th>% Full</th>
</tr>
</thead>
<tbody>
<tr>
<td>20’</td>
<td>( L )</td>
<td>(( \leq 10 ) tons)</td>
<td>( M )</td>
</tr>
<tr>
<td>50</td>
<td>40</td>
<td>10</td>
<td>1,15,26,9,</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4,5,29,23,</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3,6,14,16,</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>34,37,30,2,</td>
</tr>
<tr>
<td>38,25,18</td>
<td>28,49,39,42</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Container characteristics of the simple case study
system” chosen by the maritime company. In particular, each location (cell) is addressed by the following identifiers (see Fig. 1): (a) bay, that gives its position related to the cross section of the ship (counted from bow to stern); (b) row, that gives its position related to the vertical section of the corresponding bay (counted from the centre to the outside); (c) tier, that gives its position related to the horizontal section of the corresponding bay (counted from the bottom to the top of the ship).

Just from the resolution of this simple example we can hence note the limits of the application of a 3D-BPP algorithm for the definition of stowage plans.

3. Terminal productivity

The ever increasing number of container ship-ments is causing higher demands on the seaport container terminals, the whole container logistics chain and management, as well as the technical equipment. Consequently, the competition between seaports, especially between geographically closed ones, is increasing too. The competitiveness of a container seaport is based on different factors, such as transhipment time combined with low rates for loading and discharging. Therefore, a crucial competitive advantage is the fast turnover of the containers, which corresponds to a reduction of the berthing time and the costs of the whole transhipment process.

In what follows we examine the operations involved with the definition of master bay plans of containerships served by the Southern European Container Terminal, SECH, sited in the Port of Genoa, Italy, and analyse their impact on the quay-side productivity. In particular, we evaluate the use of a procedure based on the 3D-BPP for stowage plans in terms of terminal productivity by using two of the most important performance indexes of a generic terminal, i.e. the effective ($\sigma_M$) and mean ($\sigma_A$) net crane productivity index, given by the max-

![Fig. 3. Master bay plan configuration of the simple case study using the exact 3D-BPP algorithm.](image-url)
imum and average number of container movements per hour during the loading operations of a containership. \( \sigma_M \) and \( \sigma_A \) are expressed by Eqs. (1) and (2), respectively:

\[
\sigma_M = \frac{\sum_{i=1}^{I} \lambda_i}{\max \left\{ \sum_{j=1}^{n} t_{ij} \right\}, \quad \forall i = 1, 2, \ldots, I,}
\]

\[
\sigma_A = \frac{\sum_{i=1}^{I} \lambda_i}{\sum_{j=1}^{I} \sum_{i=1}^{I} t_{ij}/I},
\]

where \( \lambda_i \) (see Section 4, phase 2) is the total number of containers loaded by quay crane \( i \), \( i = 1, 2, \ldots, I \), and \( t_{ij} \) is the loading time necessary to position on board container \( j \), \( j = 1, \ldots, n \), by crane \( i \), \( i = 1, 2, \ldots, I \). In particular, if the number of cranes used for the stowage operations is only one, i.e. \( I = 1 \), the effective and average net crane productivity are the same (i.e. \( \sigma_M = \sigma_A \)), otherwise, if two or more quay cranes work in parallel for the same ship \( (I > 1) \), the difference between the performance quay indexes is smaller and the level of synchronisation between the quay resources is greater. In the best case, i.e. when the difference equals 0 (that is \( \sigma_M = \sigma_A \)) during the loading operations all cranes are utilised at the same level and the spare capacity of resources is minimised.

In order to compute (1) and (2) in Section 5 we make the assumption that two quay cranes are used for performing the loading operations of the ship, and compare the total loading time corresponding to the stowage plans resulting by using different heuristic approaches for MBPP. Note that the assumption of a common occurrence and, consequently, it is a reason-able and acceptable assumption to make.

### 4. A MBPP heuristic procedure for evaluating the terminal productivity

We present the main steps of an algorithm for MBPP which is an extension of the heuristic procedure given in Sciomachen and Tanfani (2003) for minimising the total loading time in the definition of stowage plans. Here we want to include in the analysis the quay cranes used for the loading operations with the aim of maximising the terminal productivity indexes introduced in Section 3. We consider the constraints described in Section 2 and use the positioning pattern, and the corresponding “enumeration” of the containers, followed in the case of 3D-BPP for “packing” subsets of containers into different portions of the ship, such that the feasibility of the solution is not violated and the net crane productivity is maximised.

The main phases of our approach are now stated.

#### 4.1. Phase 1. Bay partitioning and quay crane assignment

We first position the \( I \) available quay cranes in front of the ship. Note that, since a secure distance between cranes is required, the basic criterion followed by our procedure is to ensure a distance of at least two bays between them. In particular, starting from the first bay, we position crane \( i \), \( i = 2, \ldots, I \), every \( \lfloor b_i \rfloor + 1 \) bays, where \( b \) is the number of bays of the ship. Henceforth, let \( b_i \) be the number of bays associated with crane \( i \), \( i = 1, \ldots, I \) and \( z_i \in Z \) the first bay location available for stowing containers handled by crane \( i \).

We then assign the containers to the \( I \) cranes for allowing parallel loading operations at the quay. In this way, we define subsets \( C_i \), \( i = 1, 2, \ldots, I \) of set \( C \) of containers to be stowed in the ship by crane \( i \) in \( b_i \) bays and know the number \( \lambda_i \) of containers to be handled by crane \( i \) for evaluating our performance indices (1) and (2). We now also generate subsets \( Z_{C_i} \), \( i = 1, \ldots, I \), of all \( b_i \) bays devoted to the stowage of containers handled by crane \( i \). Note that each set \( Z_{C_i} \) is considered as an independent bin in our loading procedure (see phase 2).

Successively, by following the ship co-ordinator’s instructions, we assign containers \( C_i \), \( i = 1, 2, \ldots, I \), to predetermined groups of ship’s locations (identified by bays) among the \( b_i \) available ones according to their destination. In particular, we split set \( C_i \) into \( p \) subsets \( C_{h}^p \), \( p = 1, \ldots, P \), where \( P \) is the number of different ports visited by the ship; \( C_{h}^p \) is hence the set of containers handled by crane \( i \) having port \( p \) as final destination. All containers are now grouped together according to their destination and crane, such that \( \bigcup_{i=1}^{I} \bigcup_{h=1}^{P} C_{i}^{h} = C \) and \( C_{i}^{h} \bigcap C_{g}^{p} = \emptyset \), \( \forall h \neq g, h, g = 1, \ldots, P \) (\( \forall i \)).

Considering any slot of \( b_i \) bays we start by assigning containers with destination \( p \), \( p = 1, \ldots, P \), to each location according to stability considerations and using the following bay assignment procedure, that has been derived by adapting the procedure proposed in Ambrosino et al. (2006) for \( I = 1 \). In particular, \( \forall i \) we first assign the central bay of \( Z_{C_i} \), namely bay \( \lfloor \frac{b_i}{2} \rfloor \), to \( C_i^1 \), where \( C_i^1 \) is the subset of containers to be unloaded first; consequently, we initialise \( Z_{C_i} = \{ \lfloor \frac{b_i}{2} \rfloor \} \).
At each decision node of the main branching tree of the first assignment the size feasibility is checked as follow. If \( \chi_i^0 \), that is the number of the containers belonging to \( C_i^0 \), is not greater, within a given TEUs tolerance, than the corresponding number of locations of \( Z_{C_i^0} \), then the assignment is accepted, since no further bay is required for loading the whole set \( C_i^0 \); consequently, we start with the search of the bays for loading containers belonging to \( C_i^1 \). Otherwise, we backtrack and assign to \( C_i^1 \) also bay \((\lceil \frac{n_i}{2} \rceil + 3)\) from the right side (bow side), and consequently update \( Z_{C_i^1} \) as \( Z_{C_i^1} \cup \{\lceil \frac{n_i}{2} \rceil + 3\} \); then, we check again the feasibility of the assignment as before. In particular, if \( \chi_i^1 \) is still greater than the number of locations associated with \( Z_{C_i^1} \), we proceed by adding to \( C_i^1 \) bay \( (\lceil \frac{n_i}{2} \rceil - 3) \) from the left side (stern side). In the search for the size feasibility we continuously add, alternatively at the right and left side, the bays that are externals and contiguous to those that have been already chosen, i.e. \((\lceil \frac{n_i}{2} \rceil + 4), (\lceil \frac{n_i}{2} \rceil - 4), (\lceil \frac{n_i}{2} \rceil + 5), \) etc.; when there are no more available bay in this direction, we choose the nearest bay to the centre of partition \( Z_{C_i^1} \). When \( Z_{C_i^1} \) is large enough for loading all containers belonging to \( C_i^1 \), we accept the current assignment.

If \( P < 2 \) the whole procedure terminates. Otherwise, we assign the bays for loading containers of \( C_i^2 \) and check the feasibility of the assignment as before by computing the number of locations corresponding to \( Z_{C_i^2} \), and the bay assignment procedure will proceed for all existing subsets \( C_i^p, p = 1, \ldots, P \).

Is it worth mentioning that, for every subset \( C_i^p \), \( p = 1, \ldots, P \) we start the search for the available locations alternatively in the left and right side of the ship in order to prevent unbalances in its weight distribution during the unloading operation at the destination port \( p \).

4.2. Phase 2. Definition of the bins

The shape of a ship is different from a standard six-face solid, that is utilised as the bin in 3D-BPP. As in Sciomachen and Tanfani (2003), we split the ship into different sections in order to be able to distinguish the hold, the upper deck, the bow, the stern and some particular zones where it is not possible to stow containers. Each section can be hence considered as a bin and filled by following the main frame of the exact algorithm for the 3D-BPP. We call the biggest parallelepiped shape portion of the ship related to set \( Z_{C_i} \), \( i = 1, \ldots, I \), the “main bin” and denote it \( B_i \). Note that usually all main bins consist of about 80% of the total available stowing area (TEUs) of the ship.

Successively, \( \forall i \) we split the main bin itself into \( K_i \) identical sections, denoted “normal sections”; \( N_{ki}, k = 1,2,\ldots,K_i \), for guaranteeing the stability of the ship (see the following phases 2 and 3). Value \( K_i \) is derived from Eq. (3), where \( b_i \) and \( r_i \) are, respectively, the number of bays and rows of the main bin \( B_i \):

\[
K_i = \frac{b_ir_i}{4}.
\]  

We call the remaining portions of the ship “special sections”; they are considered and loaded separately since they correspond to (a) the lowest tier/tiers that can be too small to belong to the main section; (b) portions of the sides of the main section at different tiers; (c) bays where it is possible to stow only a few 10’ and 20’ containers. Special sections \( S_{hi}, h = 1,2,\ldots,H_i \), are numbered according to an increasing value of their tier and row, while alternating their bay index, due to stability reasons.

Note that we try to minimise the number of sections (bins) used for stowing all containers by following the criterion used by the exact 3D-BPP algorithm. Note that the minimisation of the special sections filled implies the minimisation of the total loading time (and consequently the maximisation of the average net crane productivity index), since they are generally located in different zones of the ship that could be far each other; therefore, an additional time for positioning the cranes along the quay could be required for loading containers in the special sections \( S_{hi}, h = 1,2,\ldots,H_i \).

4.3. Phase 3. Definition of the loading pattern

We balance and share out the total weight of the loaded containers and satisfy the horizontal and cross equilibrium of the ship by assigning a priori a given number of containers to each normal section \( N_{ki}, k = 1,2,\ldots,K_i \) on the basis of their weight. For simplicity, let us consider set \( C_i^p, p = 1,\ldots,P, \forall i \) and suppose that normal sections \( N_{ki}, k = 1,2,\ldots,K_i \) are devoted to the stowage of containers having port \( p \) as final destination; the same consideration holds for all the other sections.

We split set \( C_i^p \) into three subsets \( C_{i(w)}^p, w = 1,2,3, \) such that \( C_{i(1)}^p \) consists of “light” containers (up to 15 tons), \( C_{i(2)}^p \) consists of “medium” containers (ranging from 15 to 25 tons) and \( C_{i(3)}^p \) consists of
“heavy” containers (more than 25 tons). Then, we assign \( Z_{N_k} \) containers to \( N_{ki} \) such that

\[
Z_{N_k} = \sum_{w=1}^{3} \left| \frac{C'_{i(w)}}{k_f} \right|.
\] (4)

Note that \( Z_i \geq \sum_{N_k} Z_{N_k} \); if \( Z_i \) is greater we assign the remaining containers to different normal sections according to the destination and stability requirements and providing that the total weight of the containers in each pair of sections is within a given tolerance value \( \Delta \).

Recalling that the 3D-BPP algorithm starts to position the items from the left bottom corner of the bin (the origin \((0,0,0))\), and follows a vertical pattern concentrating the items near this starting point, now in each normal section \( N_{ki} \), \( k = 1,2,\ldots,K_i \) we have to find the origin point to start loading containers. We use an alternate criterion that, starting from \( N_{ki} \), \( \forall i \), determines the origin of each normal section according to the following pattern (see Fig. 4 in Section 5):

<table>
<thead>
<tr>
<th>Normal section</th>
<th>Origin</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_{1i} ): smallest bays, even rows</td>
<td>Left bottom corner</td>
</tr>
<tr>
<td>( N_{2i} ): same bays, odd rows</td>
<td>Right top corner</td>
</tr>
<tr>
<td>( N_{3i} ): next bays, odd rows</td>
<td>Left bottom corner</td>
</tr>
<tr>
<td>( N_{4i} ): same bays, even rows</td>
<td>Right top corner</td>
</tr>
<tr>
<td>etc.</td>
<td>etc.</td>
</tr>
</tbody>
</table>

Successively, through an axis rotation we consider the \( x \)-axis as the depth (instead of the width), the \( y \)-axis as the width (instead of the height) and the \( z \)-axis as the height (instead of the depth). Consequently, since the filling pattern follows a “\( y \rightarrow x \rightarrow z \)” sequence, the stowage of the containers will be first by width, then by depth and finally by height, so achieving the horizontal pattern.

In order to avoid putting smaller containers above larger ones, the containers assigned to \( N_{ki} \), \( k = 1,2,\ldots,K_i \) are sorted in an increasing order of their size and in a decreasing order of their weight, such that we choose first, for being loaded in the lowest tiers, the smallest heaviest containers. By using this loading criterion we avoid the violation of the size constraints and the presence of empty spaces between containers (that could occur by applying the 3D-BPP algorithm). Moreover, by using this ordering rule we follow the “from bottom to top” order utilised by the 3D-BPP algorithm as the loading sequence and satisfy the vertical equilibrium constraints. Note that, in this way, it is also easy to check the weight tolerance of a stack of containers.

5. Computational experiments

The proposed loading procedure has been used to solve commercial instances of MPBB. Here we present two different series of computational experiments. The first experiments regard the comparison between the solutions of 15 real instances obtained by using two strategies of the present approach and their optimal solutions obtained by solving the 0–1 Linear Programming model for MBPP reported in Ambrosino et al. (2004). The comparison is aimed at showing the effectiveness of the proposed approach in terms of the goodness of the solutions and computational time with the assumption of using one crane for the loading operation, i.e. \( \sigma_M = \sigma_A \) (see Section 3). The second series of computational experiments regard the comparison of the results of 5 instances of the same ship obtained by using, respectively, the present approach and a heuristic procedure that gives some pre-stowage rules for being able to solve the basic 0–1 Linear Programming model for MBPP presented in Ambrosino et al. (2004). Note that for the second series of results we make the assumption of using two quay cranes for the loading operations; therefore, we want to evaluate the impact of different stowage plans on terminal productivity and the use of the quay resources.

All the computational results here presented refer to stowage plans of the Chiwaua ship, that is a 198 TEUs containership located in a maritime terminal in Genoa (Italy), with 11 bays, 4 rows and 5 tiers (3 in the hold and 2 in the upper deck, respectively). Together with the ship profile that contains the information related to both structural and operational constraints, the terminal provided bay plan configuration that is useful for establishing the stowage in the available locations of the ship and for understanding its shape. Fig. 4 shows the tiers \((02,04,06,72,74)\) of the ship, the 20’ standard bays \((01,03,05,\ldots,21)\), the corresponding 40’ bays \((02,06,12,\ldots,20)\) and the rows \((04,02,01,03)\). The “dark” slots are not allowed for stowing containers (we can note that, because of the natural “slanting” shape of the hold, the lowest tiers are narrower in the external parts).

Using these documents we define the partition of the structure of the Chiwaua ship into different sec-
tions used for the stowage of the containers. As we have already said, this phase is crucially important for the goodness and feasibility of the final master bay plan. First, we identify the main bins $B_i$, $i = 1, 2$ by searching for the largest parallelepiped area; then, we split the main bins into eight normal sections (N1 $\cdots$ N8) on the basis of stability and weight considerations and the pre-processing of the destinations of the containers. The arrows depicted in Fig. 4 show the initial positions of the two quay cranes that will move in the right side during the stowage operations.

Note that in order to maximise the net crane productivity we generate 10 special sections (S1 $\cdots$ S10) consisting of lower tiers and lateral sea-side bays; in such special sections it is difficult to place containers and we enforce the stowage of 40' containers in these locations, thus reducing the number of placements in the cells requiring greater loading time. In fact, looking at Table 2 we can see an example of loading times of the chosen ship; note that the value between two contiguous locations increases when we move from the quay side (odd rows) going to the bottom (see Fig. 1), since the locations are more difficult to reach. Of course, any other linear function for the loading times can be properly assumed, provided that it depends on the location where a container is placed and on the type of quay crane used.

5.1. Comparison with the optimal solution (one quay crane)

The first series of computational experiments refer to the test instances reported in Table 3. As can be seen, the instances differ from each other

![Fig. 4. Normal and special sections of the Chiwaua ship and quay crane initial positions.](image-url)
The number of containers to load on board, ranging from 75 to 144, corresponding to a ship occupation level, in percentage, ranging from 52.66% to 100%. We increase the number of heavy containers, up to 50% of the total number of containers loaded (see instance 12), and change the size and the number of ports to be visited, that is either 2 or 3. Note that we give a 100% occupation level when 188 TEUs are loaded, since, conventionally, 10 TEU locations are operatively always let free for security and possible emergency reasons.

Table 4 reports the results of the instances of the Chiwaua ship obtained by using the 0–1 Linear Programming model for MBPP (column $L^0$) given in Ambrosino et al. (2004) and the proposed heuristic algorithm when two different partitioning strategies of the ship are used (columns I and II). All computational experiments have been performed on a PC Pentium IV.

In particular, strategy I aims at selecting the largest possible size for the normal sections, so as to reduce the 3D-BPP complexity, while strategy II...
aims at defining sections that could maximise the crane productivity indexes (1) and (2). We can determine that the average improvement from strategy “I” to strategy “II” is 1.81%, with a maximum value of 2.94% (instance 10), corresponding to about 9 minutes. Note also the differences between the optimal solution and strategies I and II, that are, respectively, 12 minutes and 8 minutes. These differences are also reflected in the values of the net crane productivity index ($\sigma_M = \sigma_A$), i.e. 22.4 and 22.7 mov./hour, respectively. The results appear to be influenced by three main factors, i.e. the occupational level of the ship, the ratio between 20’ and 40’ containers and the number of heavy container to be loaded. In fact, the lower the occupational level of the ship and the higher is the number of 40’ container, less is the improvement of the loading time between strategies I and II will be. Moreover, the smaller the number of heavy containers the greater is the average reduction of the total loading time using strategy II (see instance 10).

With reference to the computational time, it can be easily seen that in both heuristic procedures it is almost irrelevant; in fact, in all instances it is always less than 1 second. Instead, we can note how, in the case of the LP model the CPU time grows noticeably with the number of containers loaded, because of the NP-hard nature of MBPP, and is about 12 minutes on the average. It is worth mentioning that the planning office of the terminal that has provided us the data of the Chiwau containership takes from about one to one and a half hours for compiling manually the corresponding master bay plans.

5.2. Comparison between two heuristics for MBPP (two quay cranes)

The second series of results is aimed at evaluating how the loading time and the crane productivity are influenced by different stowage plans with the assumption of using two cranes for the loading operations. Table 5 gives the comparison between the solutions obtained, respectively, with the heuristics procedure for MBPP reported in Ambrosino et al. (2004) (column E) and by using the present algorithm with strategy II (column II). In the first part of Table 5 the total loading time when using only one quay crane is reported ($I = 1$) for each heuristic approach, while in the second part the given loading times are computed for each quay crane $i$, $i = 1, 2$. Note that the total loading times include the time required for quay cranes to move longitudinally along the ship during the stowage operations to allow a balanced stowage process. Therefore, the total loading times in the first scenario differ from the loading times reported in Table 4, because we have added 3 minutes and 40 seconds necessary for moving the crane alternatively in the anterior and posterior bays during the loading operations. In the second scenario, when using two quay cranes working in parallel for the same ship and positioning them in strategic starting points $z_i$ (see Section 4), the additional times for the quay longitudinal movements are respectively, 40 and 50 seconds. Consequently, the loading times in the second scenario are both less than those related to the first one ($I = 1$).

Table 6 reports the values of the net crane performance indexes (1) and (2) following the master bay plans obtained with the heuristics approaches E and II. What is important to note here is the difference between the effective and average index that, as we have already said (see Section 3) can be viewed as a proxy waste of resources during the loading operations deriving from the lack of synchronisation between the quay cranes’ work.

Note the difference between $\sigma_M$ and $\sigma_A$ decreases in all instances when using the heuristics approach

<table>
<thead>
<tr>
<th>Instance</th>
<th>Loading time ($I = 1$)</th>
<th>Loading time ($I = 2$)</th>
<th>n</th>
<th>% Full</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>II</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>3.18.42</td>
<td>3.26.28</td>
<td>1.35.25</td>
<td>1.19.24</td>
</tr>
<tr>
<td>4</td>
<td>4.15.18</td>
<td>4.25.52</td>
<td>2.27.35</td>
<td>2.08.21</td>
</tr>
<tr>
<td>7</td>
<td>4.22.24</td>
<td>4.28.16</td>
<td>2.41.38</td>
<td>2.23.34</td>
</tr>
<tr>
<td>11</td>
<td>5.23.36</td>
<td>5.28.16</td>
<td>3.29.45</td>
<td>3.16.51</td>
</tr>
<tr>
<td>15</td>
<td>6.19.54</td>
<td>6.20.28</td>
<td>4.20.05</td>
<td>3.59.30</td>
</tr>
<tr>
<td>Average</td>
<td>4.43.59</td>
<td>4.49.52</td>
<td>2.54.54</td>
<td>2.37.32</td>
</tr>
</tbody>
</table>
Table 6
Maximum and average net crane productivity index

<table>
<thead>
<tr>
<th></th>
<th>II</th>
<th></th>
<th></th>
<th>II</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>σ_M</td>
<td>σ_A</td>
<td>σ_M</td>
<td>σ_A</td>
<td>σ_M</td>
</tr>
<tr>
<td>1</td>
<td>41.90</td>
<td>45.82</td>
<td>8.55</td>
<td>42.37</td>
<td>43.86</td>
</tr>
<tr>
<td>4</td>
<td>43.81</td>
<td>46.92</td>
<td>6.64</td>
<td>43.81</td>
<td>45.21</td>
</tr>
<tr>
<td>7</td>
<td>43.67</td>
<td>46.08</td>
<td>5.24</td>
<td>44.25</td>
<td>45.05</td>
</tr>
<tr>
<td>11</td>
<td>44.93</td>
<td>46.44</td>
<td>3.26</td>
<td>45.09</td>
<td>45.59</td>
</tr>
<tr>
<td>15</td>
<td>44.04</td>
<td>45.86</td>
<td>3.98</td>
<td>45.14</td>
<td>45.71</td>
</tr>
<tr>
<td>Average</td>
<td>43.67</td>
<td>46.22</td>
<td>5.53</td>
<td>44.13</td>
<td>45.08</td>
</tr>
</tbody>
</table>

II due to the optimal work organisation of the quay cranes. We can see that even if the average quay performance indexes are greater in the bay assignment heuristics the maximum, i.e. effective, one is always better in the approach based on the 3D-BPP.

6. Conclusions

In this paper, we have presented a heuristic algorithm for MBPP based on its connection to 3D-BPP. The proposed solution method has very good performances in terms of both solution quality and computational time. In particular, the most important consideration about the performance of our heuristic algorithm is the possibility of finding stowage plans for maximising the quay terminal productivity. Moreover, our algorithm enables the loading operations of each portion of the ship to be performed in parallel thus minimising the turnaround time of the containership and hence improving the whole performance of maritime terminals.

Therefore, we believe that the proposed approach is very valuable and that one of the future directions of this research should be its application for evaluating the impact of the marine side interface in the whole organisation of the yard.

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References


